Optimisations of systems using the tensorial analysis of networks for electromagnetic compatibility

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1 Introduction

The tensorial analysis of network (TAN) developed by G.Kron in 1939 has today proved its capability to give answers to the difficult problem of the electromagnetic compatibility (EMC) prediction of complex systems[1]. The purpose of this article is to show how this technique can allow to optimize the choice of parameters in system functions. We recall briefly the principles of the TAN, before to speak of the hybrid approach. Using this last technique, we detail a very simple example to illustrate the mechanisms involved. After what we can speak about the optimization of the system studied through a classical experience plan. The techniques can be then used for many more complex systems and efficient mathematical techniques of optimizations.

2 Recall on the tensorial analysis of network

The tensorial analysis of networks (TAN) developed by G.Kron benefits today of many publications describing its application and capabilities[1, 2, 3]. G.Kron has understood all the powerfull mechanism of the tensorial algebra to resolve complex problems (at his time about electrical machines, quantum mechanics, computing, etc.). The reader can see the web description of the Kron's work in "http:\\www.quantum-chemistry-history.com\Kron_Dat\KronGabriel1.htm".
Until today the Kron's method was used by many various people in many different jobs. More than to be a method to solve the problems, the Kron's formalism is first of all a theoretical technique to establish its equations even in multiphysic[4]. The Kron's idea was to reuse as far as possible all the work made previously in modelling by an engineer in any new problem he had to solve. To do that, he shows that the TAN gives all the mathematical tools needed to connect old networks in a new one, or to make the inverse, i.e. to cut a complex problem in littleler ones, without missing any information from the global network. Kron has called this second possibility "Diakoptic"[5].
The Kron's method is based on the use of two dual spaces: the voltage space and the current one, each of them defined in R^n. Whatever is the representation of a network (using edges, vertices, nodes, meshes, ...), Kron had remarked that its total energy should not change. So, the global power of any network can be seen as an invariant.

Once two dual spaces and an invariant are defined, next step is to define a metric[6]. In case of electrical circuit, this metric is not a Riemannian one. But its stills possible to define it through operators, and the result is a tensorial algebra to theoretically study any complex networks[3]. The technique used is so a real topological one as deeply shown by Branin[7]. In all the article we use the index notation defined by Einstein[8].
On the principle, once defined the covector of the effort (voltage in an electrical circuit) e^{μ} and the flux (current) vector f^{ν}. The invariant W results from their contracted product $W = e_{\mu} f^{\mu}$. The metric can be deduced from the same invariant writing: $W = z_{\mu\nu} f^{\mu} f^{\nu}$. Classically, the electrical circuit solvers work in the nodal description. They often use the Modified Nodal Analysis which includes the voltage sources, but it doesn't change the fundamental concepts[1]. It means they translate the impedance matrix in the nodal space using the incidence matrix[9]. This matrix can be seen as a connectivity between the voltage differences V defined in the edge space, and the electrical potentials ψ defined in the nodes space. Starting from the gradient Maxwell's law we can simply write: $V_a = B_a^p \psi_p$. Kron has shown that this kind of translation could be used between the currents f defined in the edge space and the meshes currents m. To do that, he define a connectivity C writing: $f^a = C_{\mu}^a m^{\mu}$. This transformation defined, it began possible to compute all the problem in the mesh description, changing the metric through: $z_{\mu\nu} = C_{\mu}^a C_{\nu}^b z_{ab}$. What is remarkable, is the fact that in this space, the edge voltage vanishes. Effectively, as the Maxwell's equation tell us, $Rot(Grad\mathbf{V}) = 0$. The whole problem is simply compacted in a unique equation: $e_{\mu} = z_{\mu\nu} m^{\nu}$. But for various reasons, it can be interesting to keep a nodal description, at least to reuse cases already computed under this space for example. To allow this mixted approach, the authors have proposed to create mix tensors[10]. This mechanism give all the agility needed to manage the complexity and the diversity in systems.

[1]More, our hybrid formalism get out this problem as we will show it in the example.

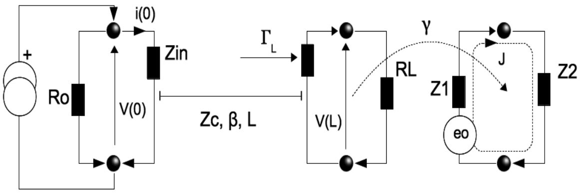

Figure 1: System considered as illustration

3 A simple example as illustration

We consider the system described fig. 1. All the next equations used are extracted from Paul's formalism on lines[11].

The current in the input of the line can be defined depending on the incident voltage wave and the coefficient reflexion Γ_L:

$$i(0) = \frac{V^+}{Z_c}\left[1 - \Gamma_L e^{-2j\beta L}\right] \qquad (1)$$

But $i(0)$ is a proportion of the current source i_0:

$$i(0) = i_0 \frac{R_0}{R_0 + Z_{in}} = \alpha i_0 \qquad (2)$$

V^+ is defined as a function of the voltage at the input of the line:

$$V^+ = \frac{V(0)}{1 + \Gamma_L e^{-2j\beta L}} \qquad (3)$$

Combining the previous equations we obtain:

$$V(0) = Z_c \alpha i_0 \left(\frac{1+\Gamma_L e^{-2j\beta L}}{1-\Gamma_L e^{-2j\beta L}} \right) \quad (4)$$

On the other side of the line we can define the relation between the transmitted voltage and the input voltage and as a consequence, the relation between the transmitted current and the input voltage:

$$\begin{aligned} &V(L) = V(0) e^{-j\beta L} \tfrac{1+\Gamma_L}{1+\Gamma_L e^{-2j\beta L}} \dots \\ &\dots \Rightarrow i(L) = \tfrac{1}{R_L} V(L) \end{aligned} \quad (5)$$

Writing:

$$Y = \frac{1}{\alpha Z_c} \left(\frac{1-\Gamma_L e^{-2j\beta L}}{1+\Gamma_L e^{-2j\beta L}} \right) \quad (6)$$

The nodal relation is synthetized in $i_0 = YV(0)$.
On the second network, the mesh relation is:

$$e_0 - j\gamma\omega i(L) = (Z_1 + Z_2) J = ZJ \quad (7)$$

Where γ is the mutual inductance coefficient. It is there assumed that the back interaction from the second network to the first one is negligeable. Using the relation between $i(L)$ and $V(0)$ we can write: $i(L) = gV(O)$. Finally the whole system can be written:

$$\begin{bmatrix} i_0 \\ e_0 \end{bmatrix} = \begin{bmatrix} Y & 0 \\ j\gamma g\omega & Z \end{bmatrix} \begin{bmatrix} V(0) \\ J \end{bmatrix} \quad (8)$$

This is the basic principle of hybrid tensor applied to connected networks. This can be applied to large networks where the previous elements become vectors and the metric is hybrid with both impedance, admittance metrics and matrix too making the connection between the element of same physical dimension. The previous equation becomes:

$$\begin{bmatrix} i^a \\ e_\mu \end{bmatrix} = \begin{bmatrix} Y^{ab} & M^a_\nu \\ M^b_\mu & Z_{\mu\nu} \end{bmatrix} \begin{bmatrix} V_b \\ J^\nu \end{bmatrix} \quad (9)$$

4 System optimization

Writing $\zeta = j\gamma g\omega$, $\lambda = Y^{-1}$ and $S = Z^{-1}$, the solution of the previous system is given by:

$$\begin{bmatrix} V(0) \\ J \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ -\zeta\lambda & S \end{bmatrix} \begin{bmatrix} i_0 \\ e_0 \end{bmatrix} \quad (10)$$

which can be compacted in the form: $f^\nu = \Omega^\nu{}_\mu h^\mu$. Now we wonder how J depends on γi_0 or e_0 in frequency, because we hope to keep the coupling influence (γi_0) inferior to the self generator e_0. We can theoretically calculate for the first request:

$$\left| \frac{\partial J}{\partial \gamma} \right| = \left| \frac{\partial f^2}{\partial \gamma} \right| = \left| \frac{\partial}{\partial \gamma} \left(-\zeta\lambda i_0 + S e_0 \right) \right| \quad (11)$$

We found easily: $\left| \frac{\partial J}{\partial \gamma} \right| = |g\omega\lambda|$. The second request is directly given by $|S|$. The first term gives:

$$\begin{aligned} &\left| \tfrac{\partial J}{\partial \gamma} \right| = \tfrac{Z_c R_0 (1+\Gamma_L)}{(Z_c + R_0) R_L} \omega \dots \\ &\dots \left\{ \sqrt{[1 + \Gamma_0 \Gamma_L \cos(2\beta L)]^2 + [\Gamma_0 \Gamma_L \sin(2\beta L)]^2} \right\}^{-1} \end{aligned} \quad (12)$$

With Γ_0 the coefficient of reflexion: $\frac{Z_c - R_0}{Z_c + R_0}$.
So, to keep the dependencies equals in a matched line case, it is necessary to have the admittance S of the form: $\left[R\sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2}\right]^{-1}$. Until the cutoff frequency ω_0, the generated current will keep the same distance to the one induced by i_0.

5 Conclusion

Through this simple example we try to illustrate the method and how it can comply with the various needs of any engineers. The optimization of the system can be studied numerically of course, through experience plans[12]. Once an equation is available to represent the system, any analytical mathematical study can be imagine to realize a theoretical analysis of the system behavior. Several studies was already conducted using this technique in conjonction with uncertainties computations. Very good results were obtained showing that the Kron's method really gives a support for these kinds of complex problems, allowing all the formulations to take part of its formalism.

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